Distancias Relativas II

1) distancia ponto a plano

$$d(\bar{P}, \pi) = \frac{|a\bar{x} + b\bar{y} + c\bar{z} + d|}{\sqrt{a^2 + b^2 + c^2}}$$

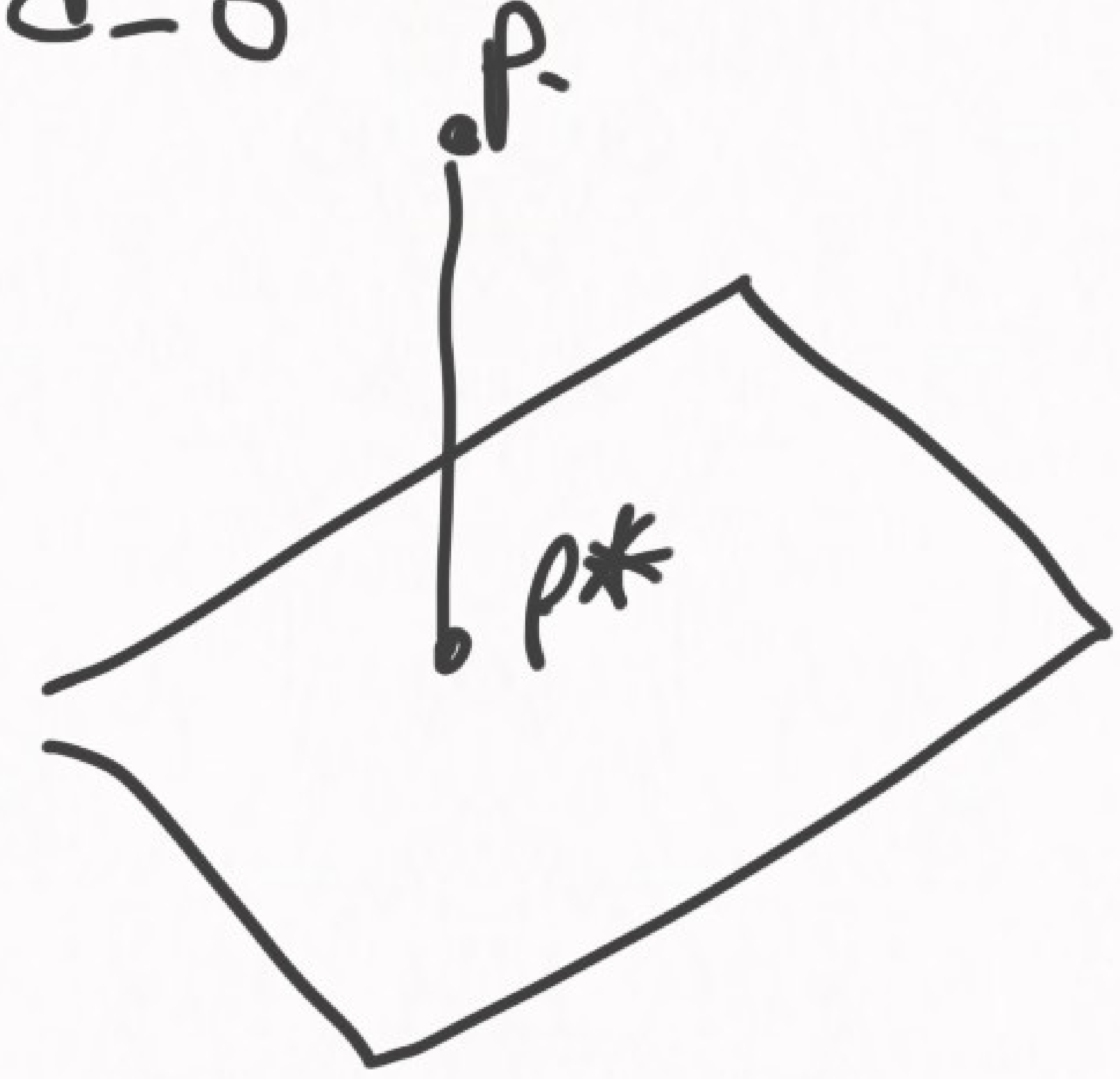
$\bar{P} = (\bar{x}, \bar{y}, \bar{z})$

$\pi: ax + by + cz + d = 0$

Achar P^* tq

$\downarrow$

ponto que realiza a distancia

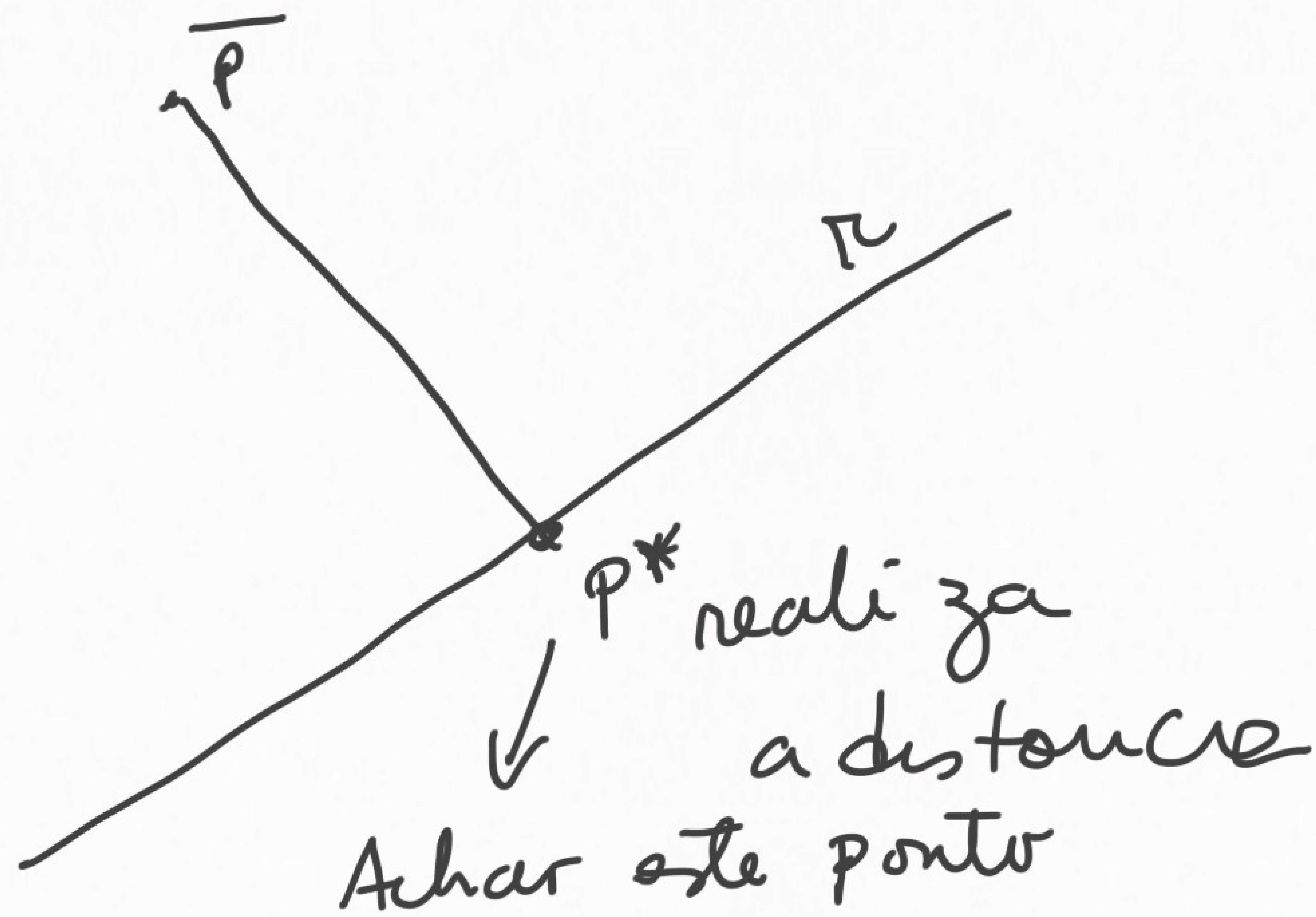


2) distancia de ponto a reta

$$d(\bar{P}, r) = \frac{\| P_r\bar{P} \times v_r \|}{\| v_r \|}$$

$r: P_r, v_r$ ↳ vetor diretor

↑ponto



3) distancia reta a plano

$\pi : ax+by+cz+d=0 \Rightarrow n_\pi=(a,b,c)$

$r : P_r$, $v_r \to$ vetor diretor

$P_r \downarrow$ ponto

$$\langle n, v_r \rangle = \begin{cases} \neq 0 \Rightarrow d(r,\pi)=0 \ \text{intesecção} \\ = 0 \Rightarrow d(P_r,\pi) = \begin{cases} 0 & r \subset \pi \\ \neq 0 & \text{dista} \ \ r \parallel \pi \end{cases} \end{cases}$$

4) distancia entre planos

$\pi \quad ax + by + cz + d = 0$

$\alpha \quad \bar{a}x + \bar{b}y + \bar{c}z + \bar{d} = 0$

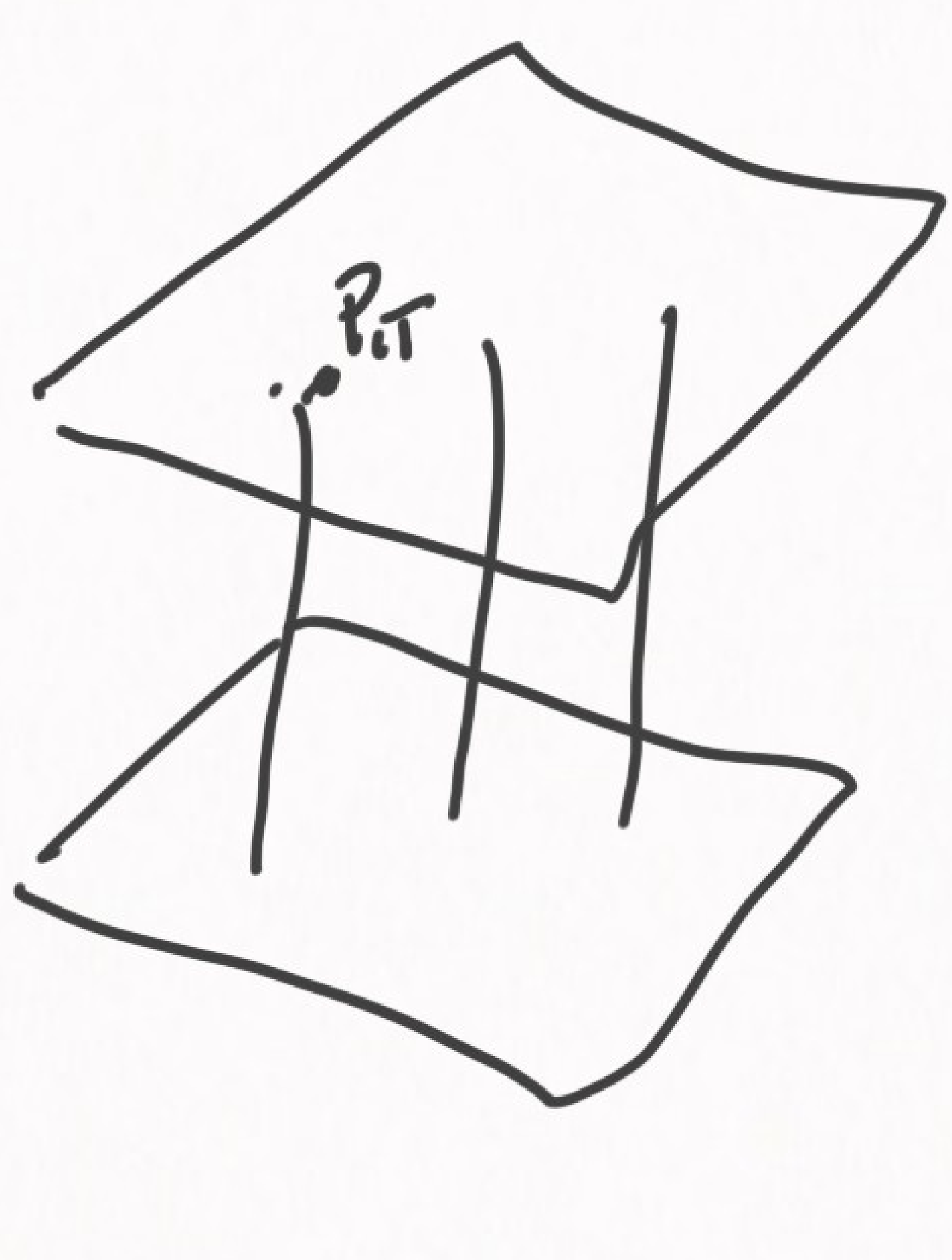
$n_\pi = (a, b, c) \qquad n_\alpha = (\bar{a}, \bar{b}, \bar{c})$

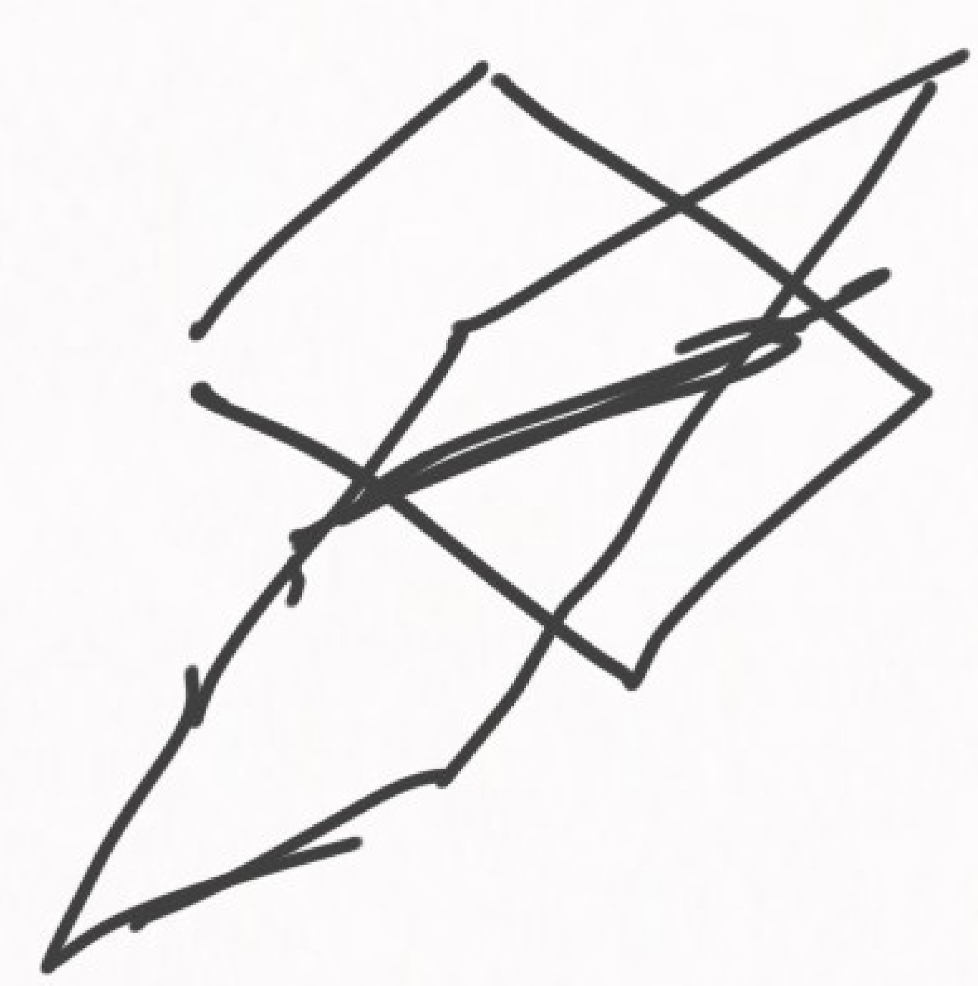
$$n_\pi \times n_\alpha = \begin{cases} 0 & \text{planos } // \\ \neq 0 & d(\pi, \alpha) = 0 \\ & \text{concorrentes} \end{cases}$$

$= 0$

Achar $P_\pi \in \pi$

$$d(P_\pi, \alpha) = \begin{cases} = 0 & \text{mesmo plano} \\ \neq 0 & d(\pi, \alpha) \neq 0 \\ & \text{e planos } // \end{cases}$$





$n_\pi \times n_\alpha \neq 0$ Achar a reta interseção

$$\begin{cases} ax + by + cz = -d \\ \bar{a}x + \bar{b}y + \bar{c}z = -\bar{d} \end{cases}$$

→ inf. soluções ⇒ escrever estas inf sol de forma paramétrica fornecia eq da reta

como achar um ponto do plano

$x = y = 0 \Rightarrow$ calcular $\bar{z} \Rightarrow (0, 0, \bar{z})$

$y = z = 0 \Rightarrow$ calcular $\bar{x} \Rightarrow (\bar{x}, 0, 0)$

$x = z = 0 \Rightarrow$ calc $\bar{y} \Rightarrow (0, \bar{y}, 0)$

distancias entre retas

$r: \quad P_r \quad v_r \Rightarrow \begin{cases} x = P_{r1} + t\, v_{r1} \\ y = P_{r2} + t\, v_{r2} \\ z = P_{r3} + t\, v_{r3} \end{cases}$

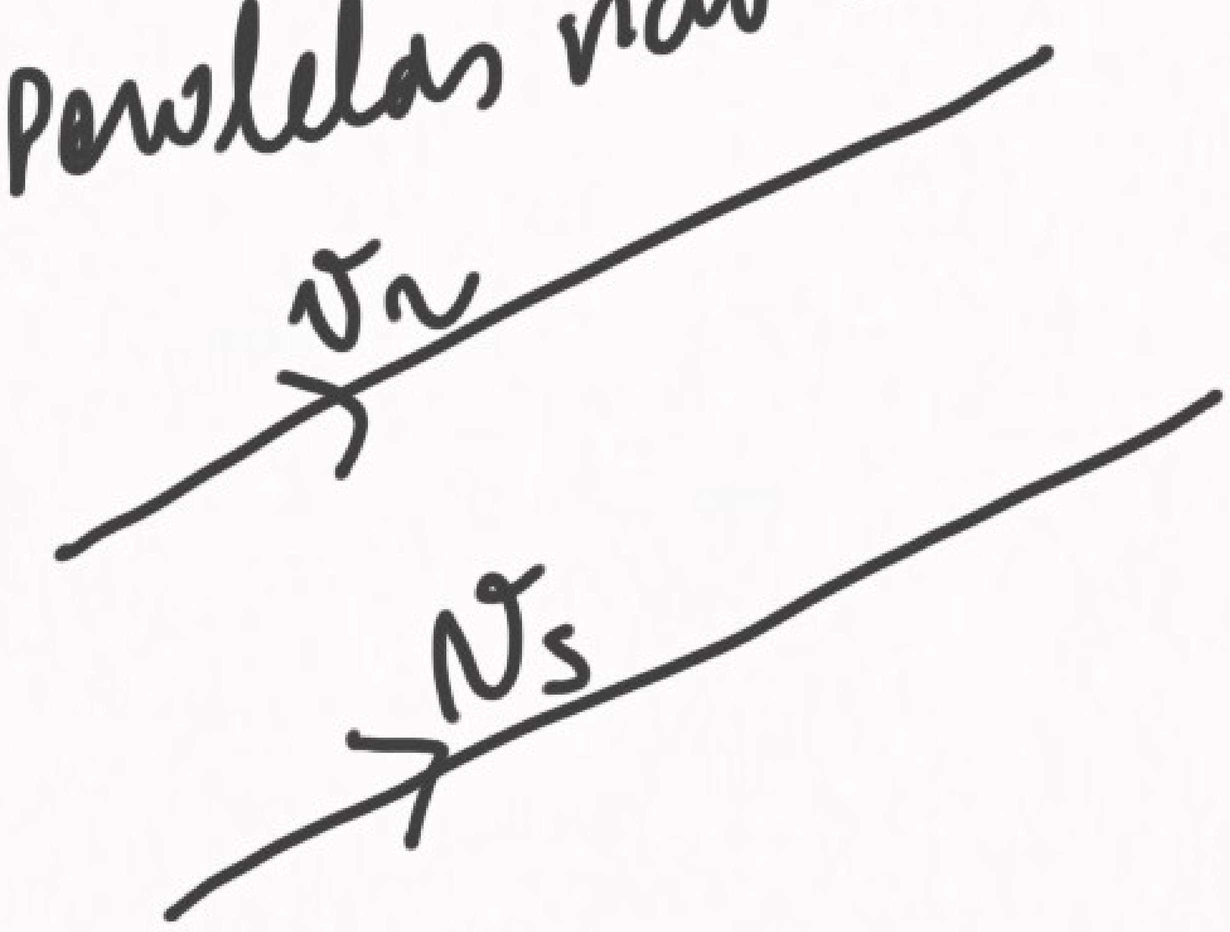
$s: \quad P_s \quad v_r$

$\hookrightarrow \begin{cases} x = P_{s1} + t\, v_{s1} \\ y = P_{s2} + t\, v_{s2} \\ z = P_{s3} + t\, v_{s3} \end{cases}$

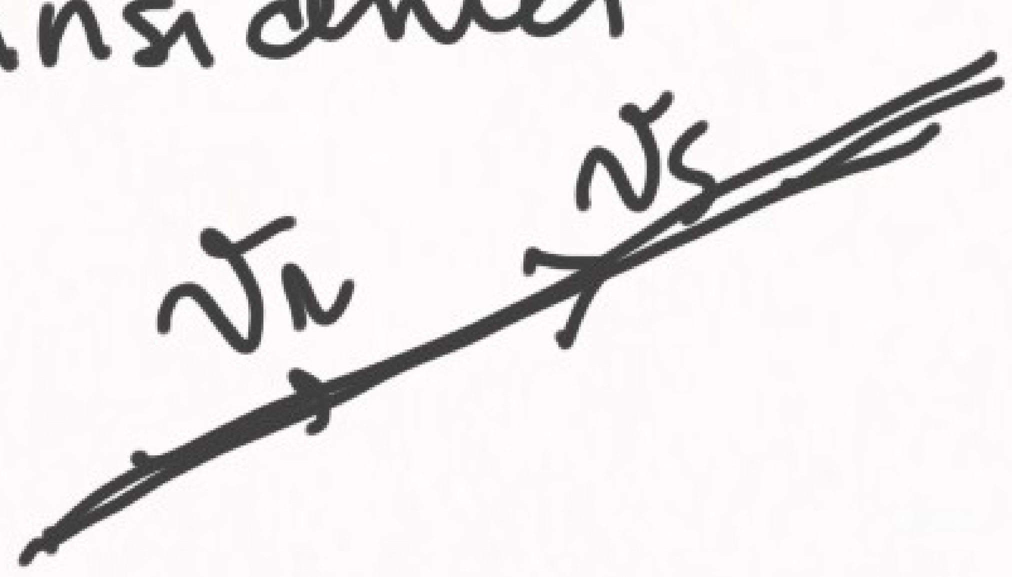
a) $\boxed{v_r \times v_s = 0}$

paralelas não coinsidentes

a1)

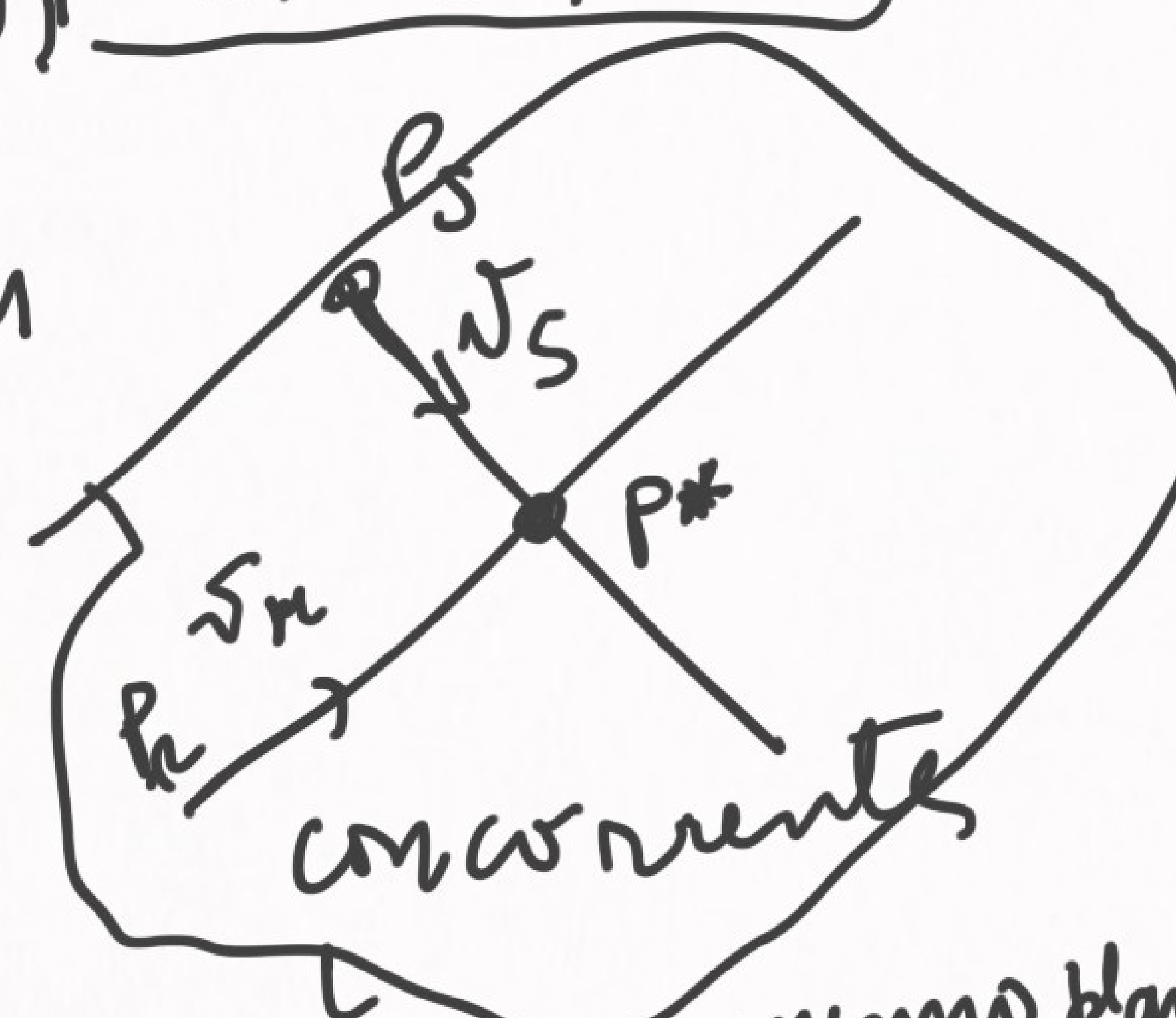


a2) coinsidentes

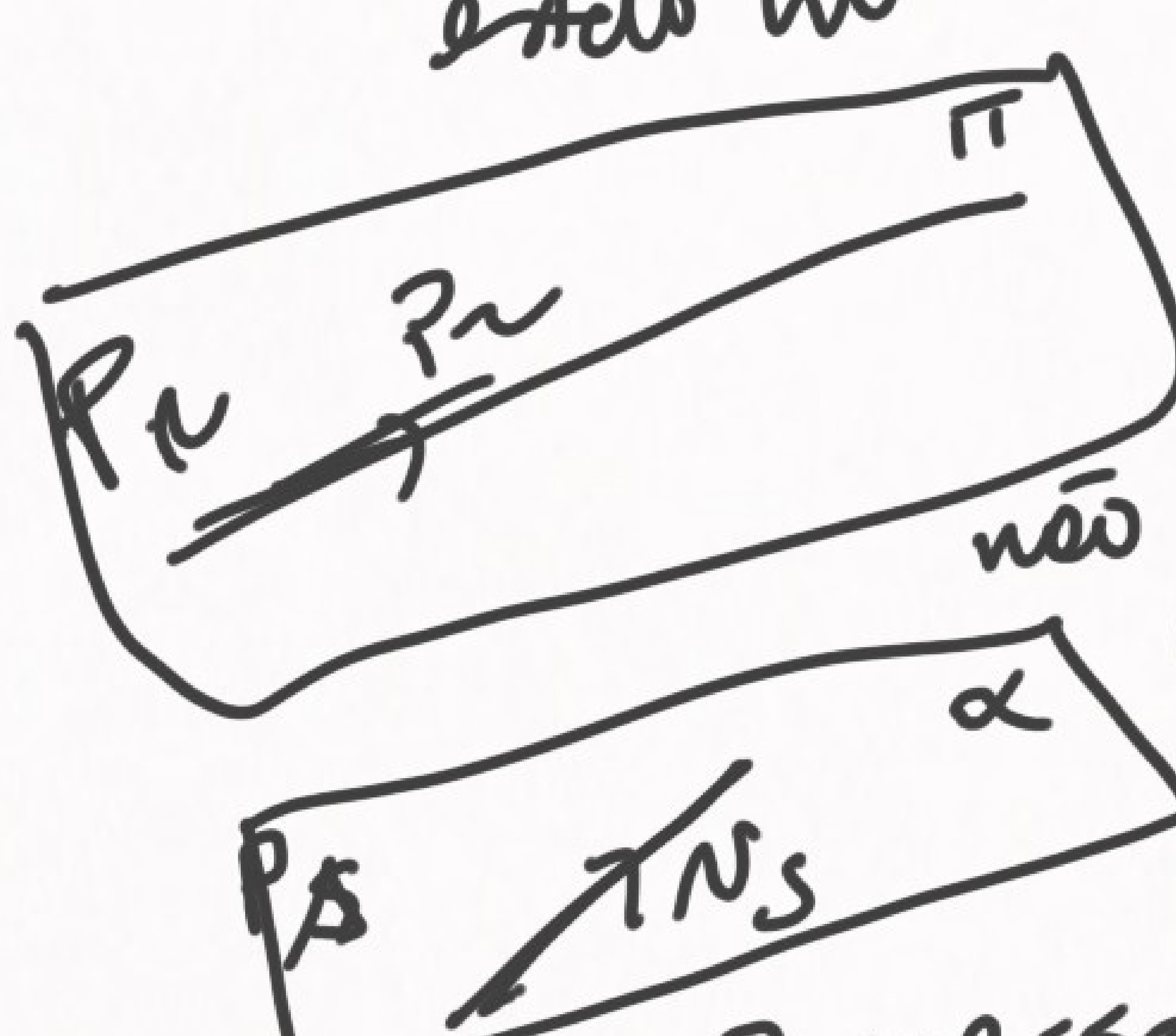


b) $\boxed{v_r \times v_s \neq 0}$

b1



estão no mesmo plano



não tem interseção

Reversas

a) $\underline{v_r \times v_s} = 0$

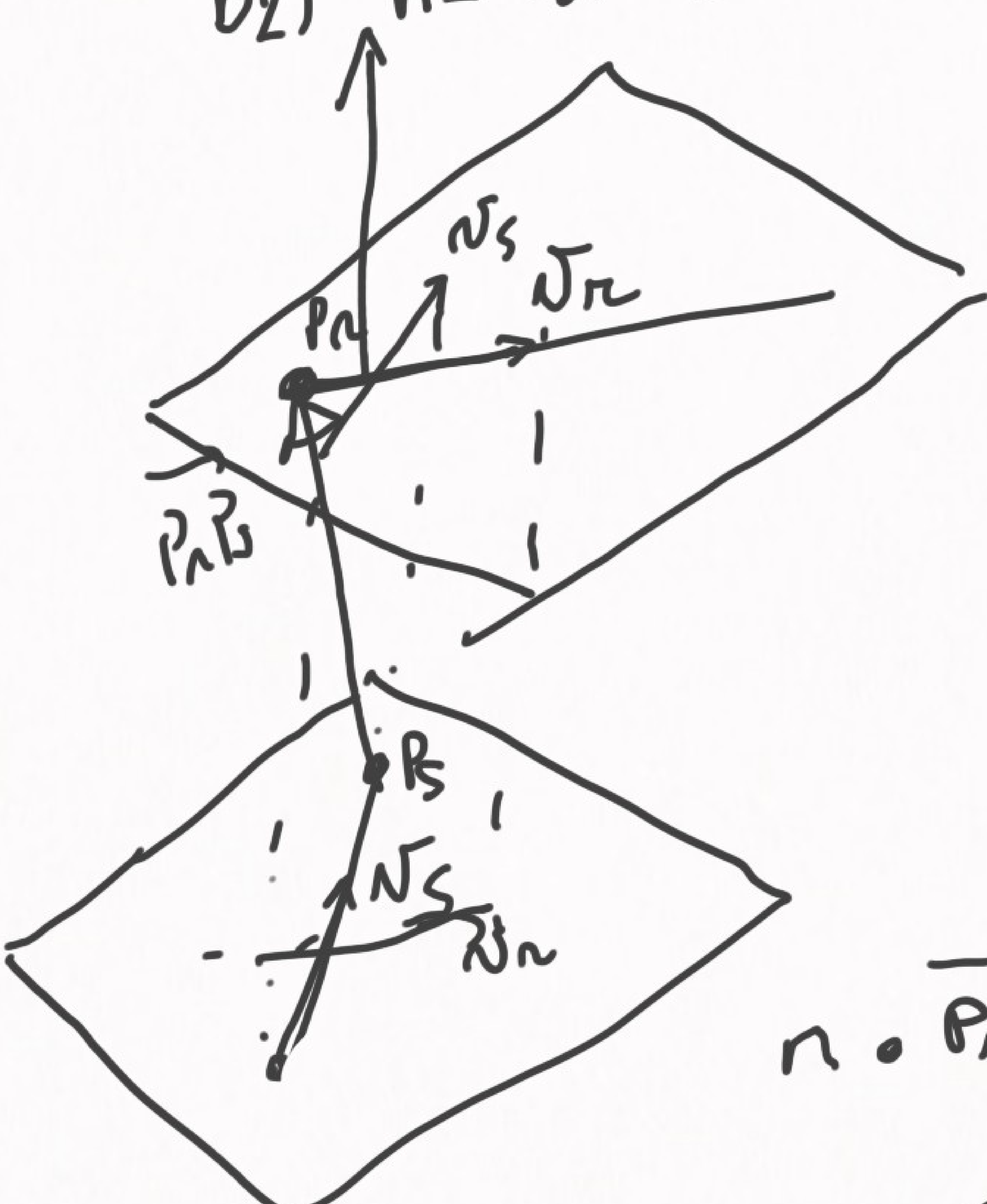
$d(P_r, s) = \begin{cases} = 0 & \text{Paralelas coinsidentes} \\ \neq 0 & \text{Paralelas não considente} \end{cases}$

reta

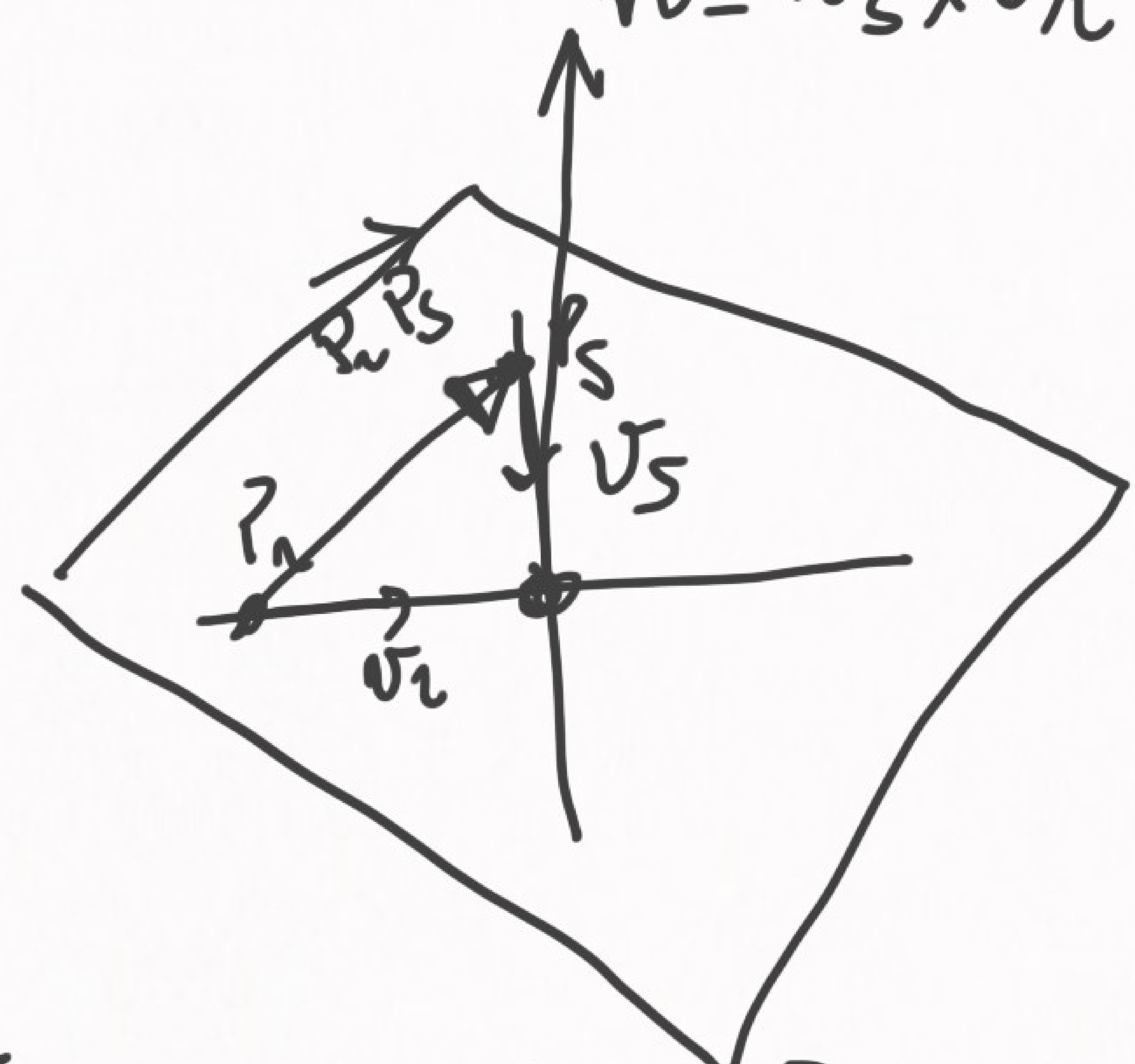
$d(r,s) = d(P_r, s)$

b)

b2) $\vec{n} = v_s \times v_r$



b1) $\vec{n} = v_s \times v_r$



$n \cdot \overrightarrow{P_rP_s} = (v_r \times v_s) \cdot P_rP_s =$

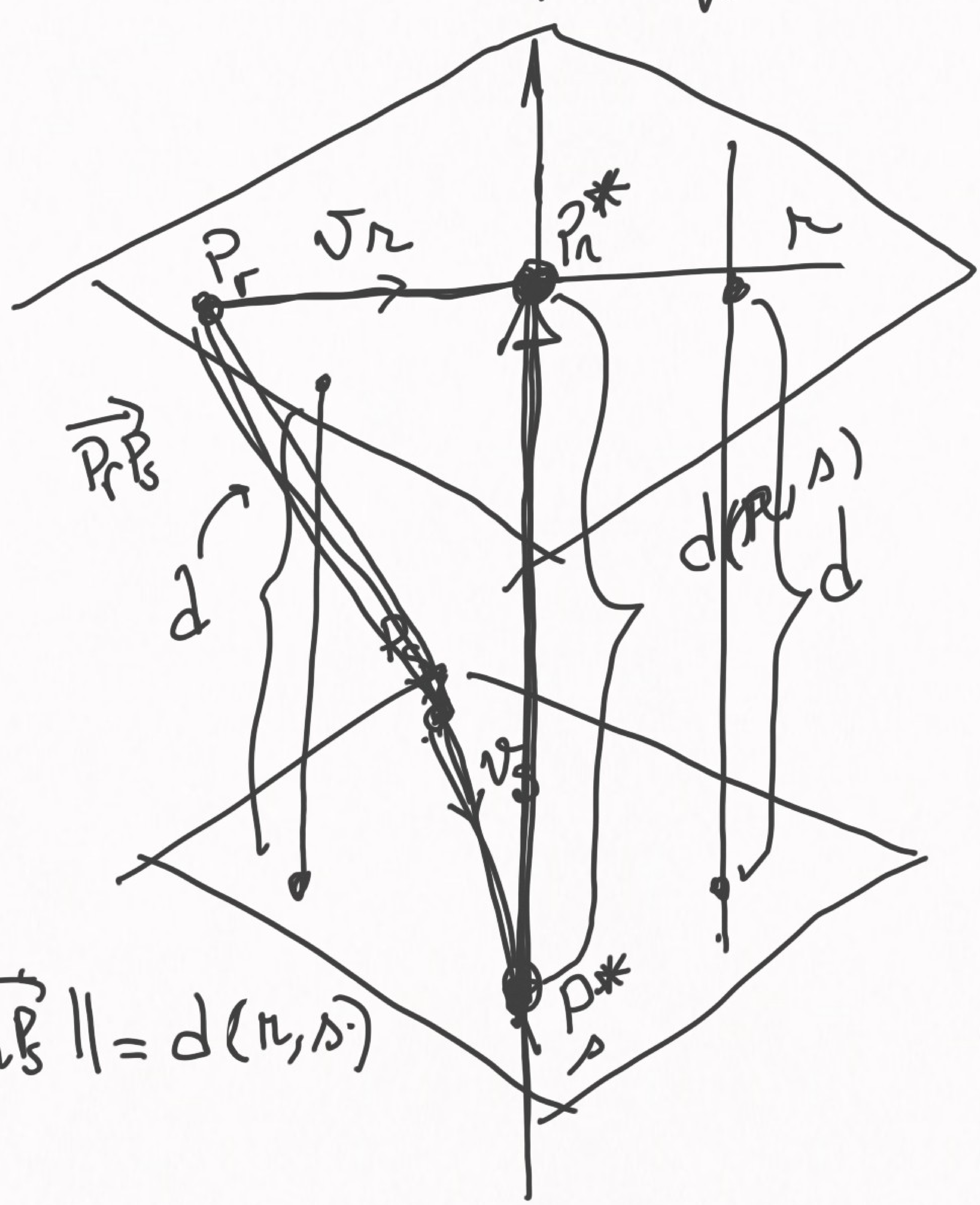
$= 0$ concorrente /coplanares

$\neq 0$ reversas

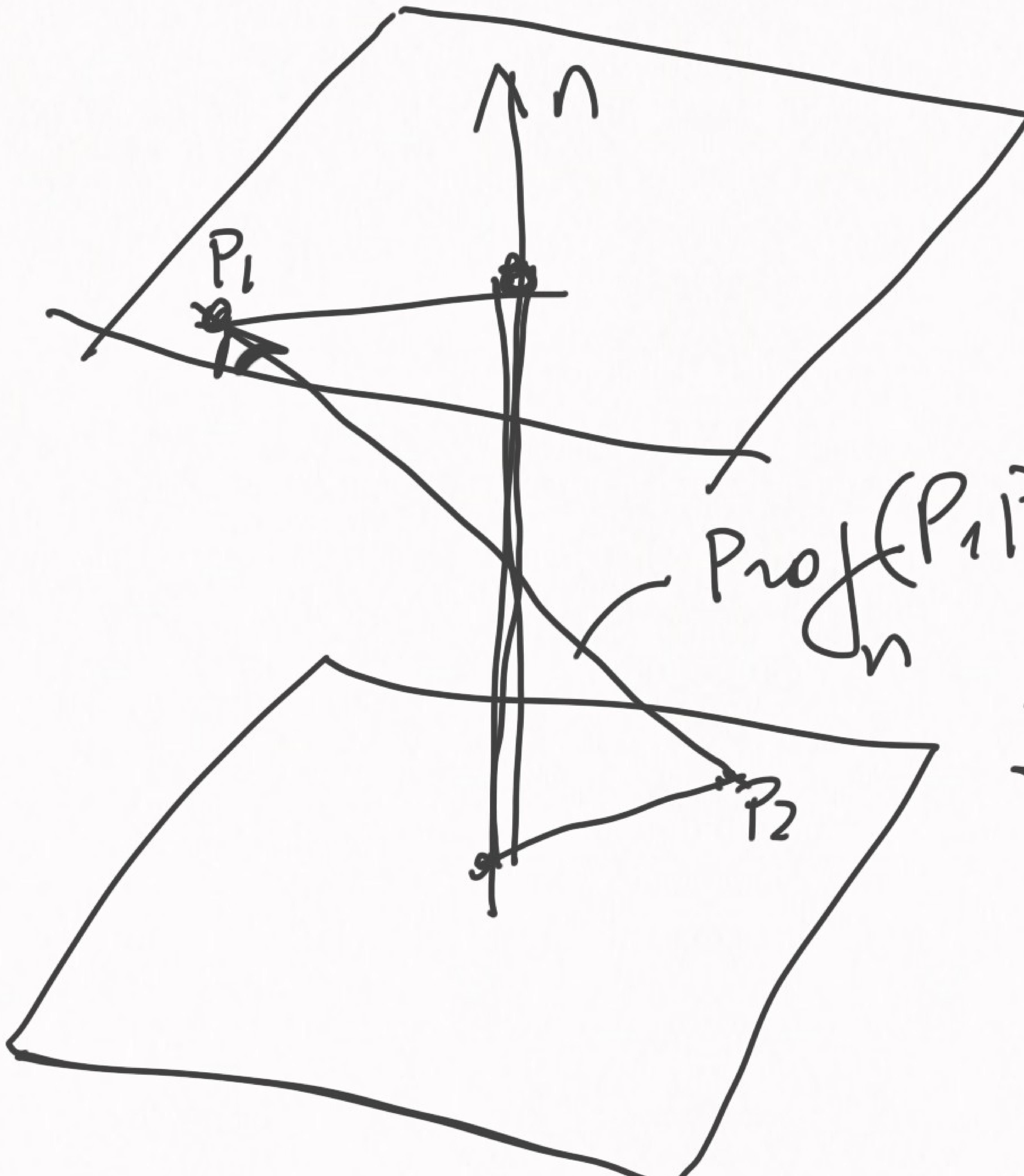
Calculo da distancia

b1) $d(r,s) = 0$
coinsidentes

b2) Reversas



$$\|\text{Proj}_n \overrightarrow{P_rP_s}\| = d(r,s)$$

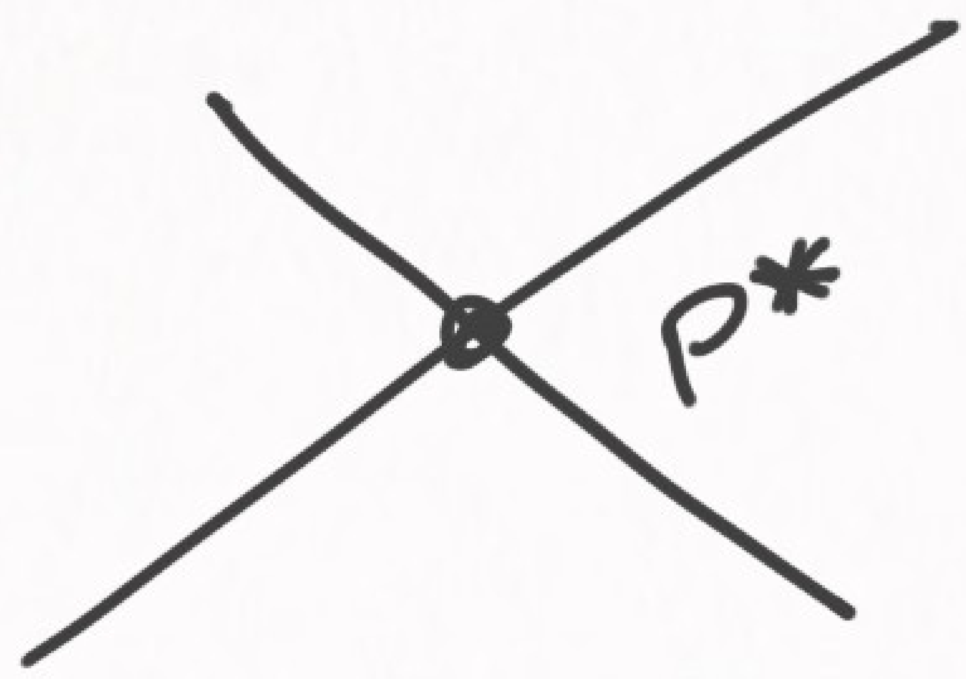


$\mathrm{Proj}_n(P_1P_2)$ = distância entre os planos

$$d(r,s) = \left\| \mathrm{Proj}_{\vec{n}} \overrightarrow{P_rP_s} \right\| = \frac{\left| \vec{n} \cdot \overrightarrow{P_rP_s} \right|}{\|\vec{n}\|} =$$

$$= \boxed{\frac{\left| (v_r \times v_s) \cdot \overrightarrow{P_rP_s} \right|}{\| v_r \times v_s \|}} = \begin{cases} 0 & \text{coinsidentes} \\ \neq 0 & \text{Reversas} \Rightarrow d(r,s) \end{cases}$$

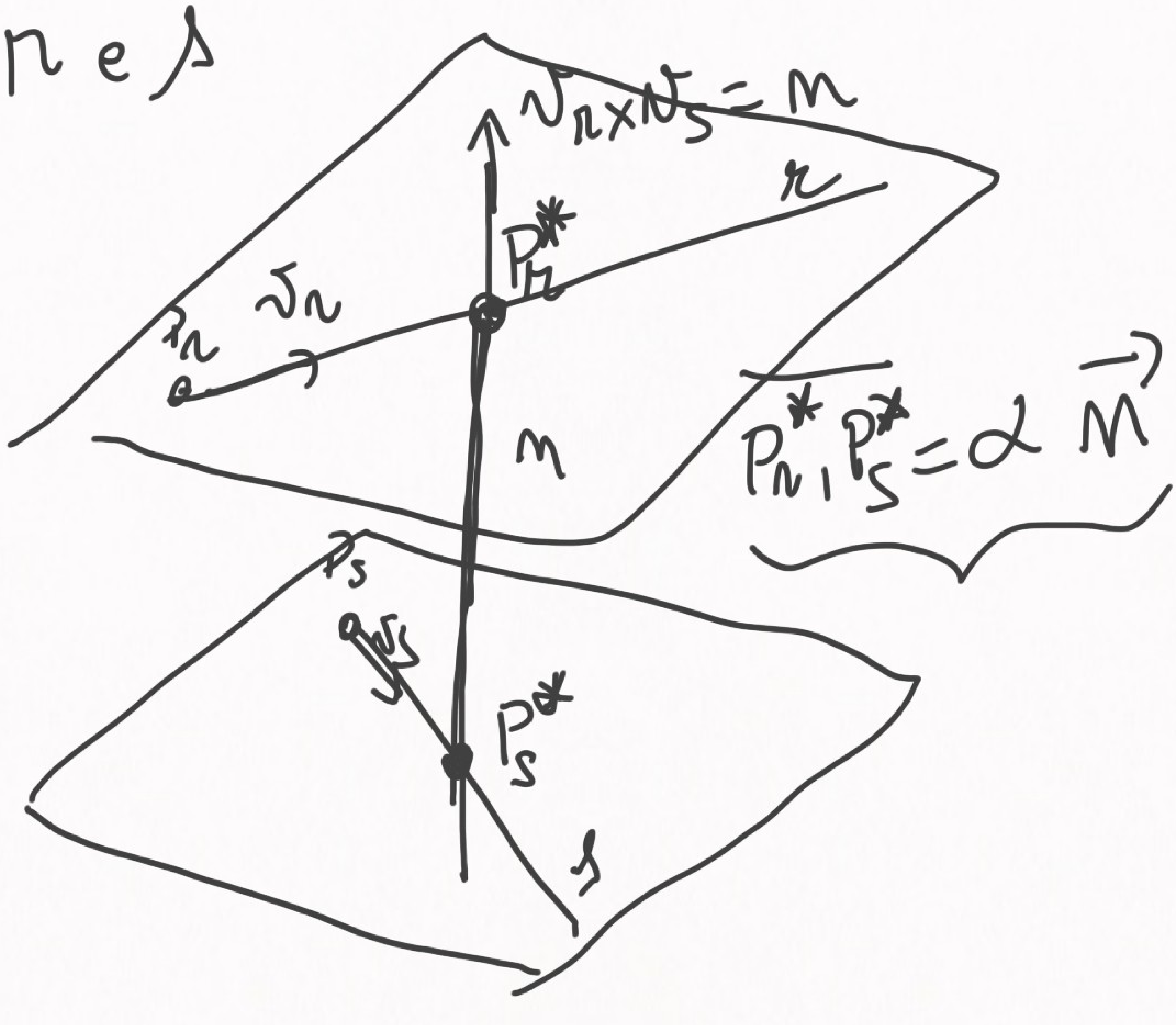
1) concorrentes



ponto de concorrencia

2) Reversas

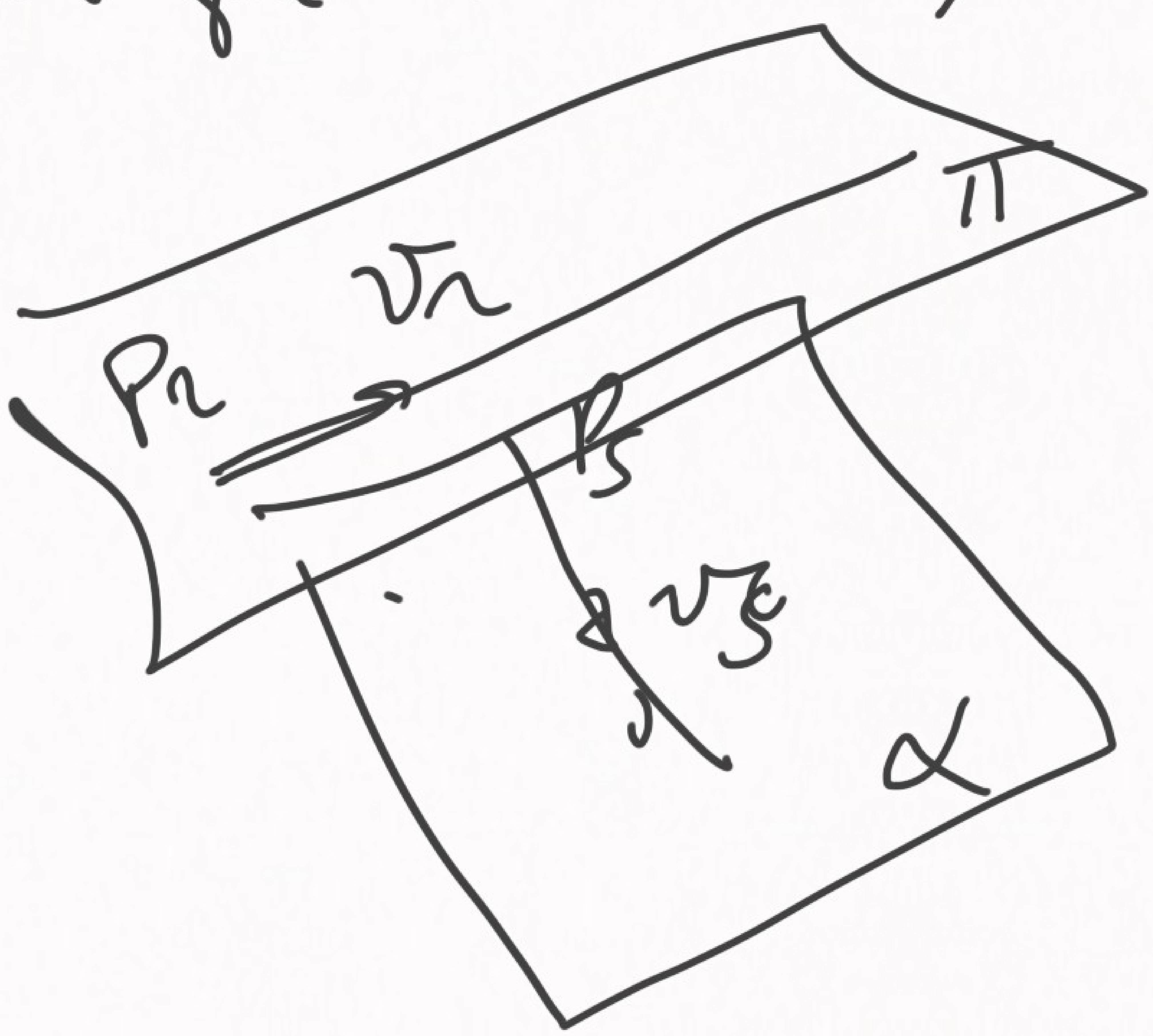
Achar a reta que é ortogonal a r e s, e intersecta a r e s



3) retas são reversas

Achar os planos paralelos π, α

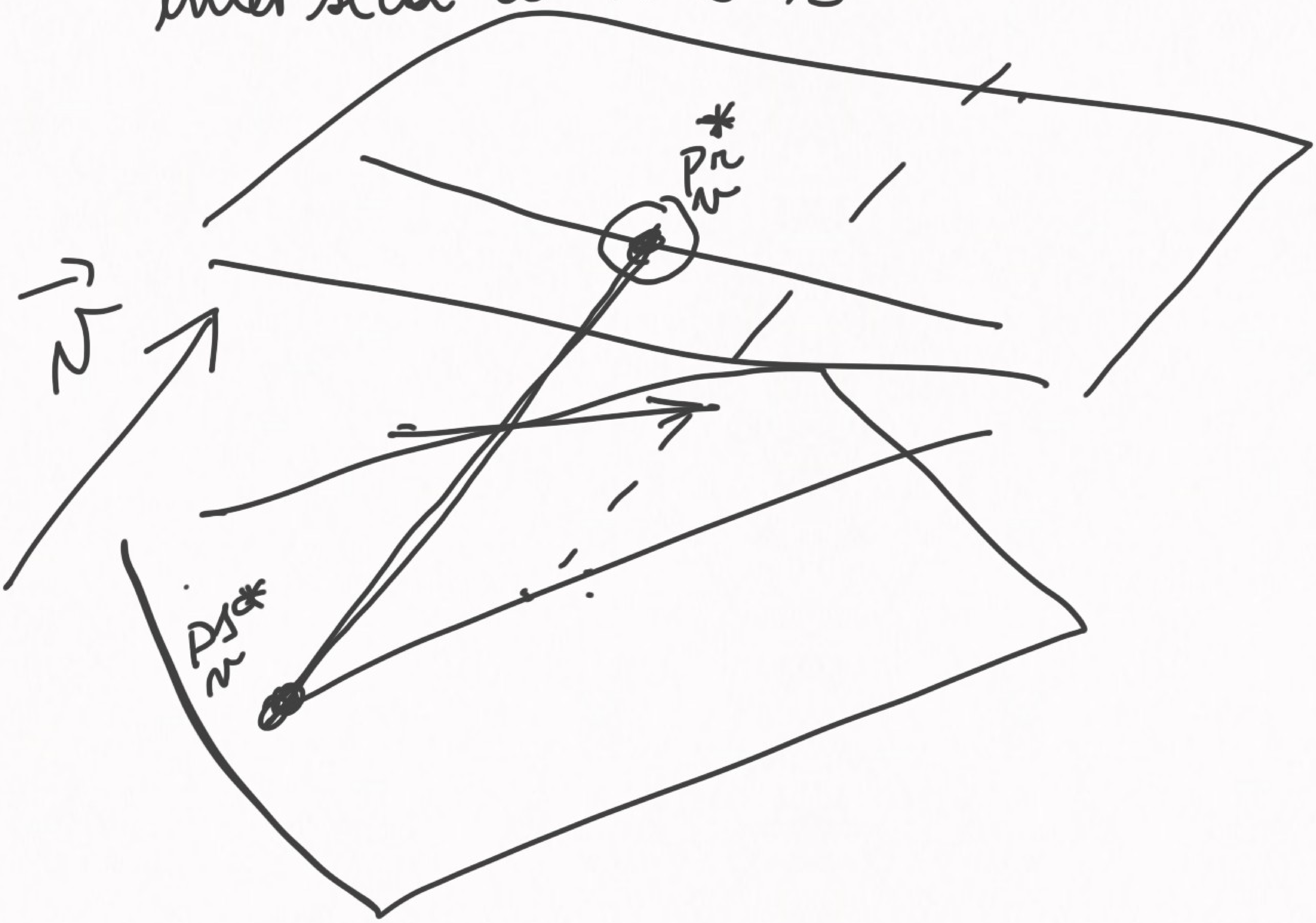
tal que $r \subset \pi$ e $s \subset \alpha$



$d(\pi, \alpha) = d(r, s)$

4) Achar $\boxed{P_r^* \text{ e } P_s^*}$

5) Achar reta // $\vec{v}$ que interseca a r e s



Exercicio

dada r $\dfrac{-x-1}{-1} = \dfrac{y+3}{2}; z = 1$

e s $\begin{cases} x = 2 + t \\ y = -t \\ z = t \end{cases}$

Hoje
Achar as posicoes relativas de r, s
se d(r,s) e se são concorrentes
Achar Ponto de concorrencia
se são reversas Achar $P_r^* P_s^*$

Achar a reta paralela a
$\boxed{v = (1,0,1)}$ tal que intersecta
a r e s Tarefa para casa

$$\frac{-x-1}{-1} = \frac{x+1}{1} = x+1$$

$\hookrightarrow r$

$$\boxed{\frac{x+1}{1} = \frac{y+3}{2}, \; z = 1}$$

$r: \quad P_r = (-1,-3,1) \qquad v_r = (1,2,0)$

$s: \quad P_s = (2,0,0) \qquad v_s = (1,-1,1)$

Posição relativa

$$v_r \times v_s = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 1 & -1 & 1 \end{vmatrix} = 2\hat{i} - \hat{j} - 3\hat{k}$$

$\vec{M} = (2,-1,-3) \neq 0$

$\Rightarrow$ concorrentes ou Reversas

$\overrightarrow{P_rP_s} = (3,3,-1)$

$$\underbrace{(2,-1,-3)}_{(v_r \times v_s)} \cdot \underbrace{(3,3,-1)}_{\overrightarrow{P_r P_s}} = 6-3+3 = 6 \neq 0$$

Reversas

$$d(r,s) = \frac{6}{\sqrt{2^2+1^2+3^2}} = \frac{6}{\sqrt{14}}$$

Achar P_r^* e P_s^*

$$\begin{cases} P_r^* \in r \\ P_s^* \in s \\ \overrightarrow{P_r^* P_s^*} = \alpha \vec{n} \end{cases}$$

$$P_r^* = (-1,-3,1) + t^*(1,2,0) = (-1+t^*, -3+2t^*, 1)$$

$$P_s^* = (2,0,0) + \ell^*(1,-1,1) = (2+\ell^*, -\ell^*, \ell^*)$$

$$\overrightarrow{P_r^* P_s^*} = (3+\ell^*-t^*, -\ell^*+3-2t^*, \ell^*-1) = \alpha(2,-1,-3)$$

$$\begin{cases} 3 + l^* - t^* = 2\alpha \\ -l^* - 2t^* + 3 = -\alpha \\ l^* - 1 = -3\alpha \end{cases}$$

sistema 3 eq 3 incó

Transição

$$\begin{array}{l} (1)+(2) \\ (2)+(3) \end{array}\begin{cases} 6 - 3t^* = \alpha \\ -2t^* + 2 = -4\alpha \end{cases}$$

$\Rightarrow \alpha = 6 - 3\frac{13}{7} = \frac{3}{7}$

$l^* = 1 - 3\frac{3}{7} = \frac{-2}{7}$

$$-2t^* + 2 = -4(6 - 3t^*)$$

$$-t^* + 1 = -12 + 6t^*$$

$$13 = 7t^* \Rightarrow$$

$$\boxed{t^* = \frac{13}{7}}$$

$$P_r^* = \left(-1 + \frac{13}{7}, -3 + 2\left(\frac{13}{7}\right), 1\right) = \left(\frac{6}{7}, \frac{5}{7}, 1\right)$$

$$P_s^* = \left(2 + \left(-\frac{2}{7}\right), -\left(-\frac{2}{7}\right), -\frac{2}{7}\right) = \left(\frac{12}{7}, \frac{2}{7}, -\frac{2}{7}\right)$$

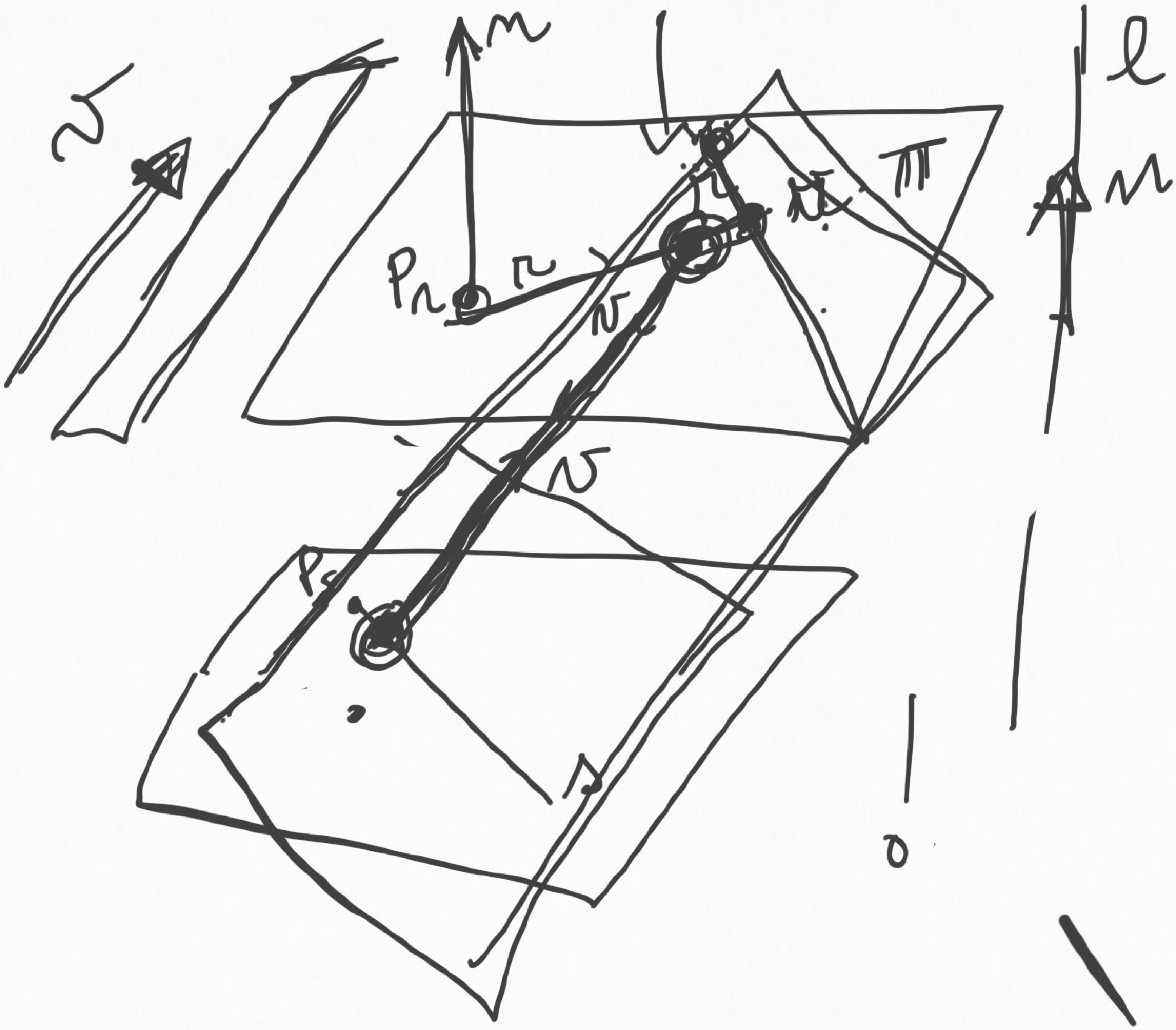
Achar a reta que Passa por P_r^* e P_s^*

$$\overrightarrow{P_rP_s} = \left(\frac{12}{7} - \frac{6}{7}, \frac{2}{7} - \frac{5}{7}, -\frac{2}{7} - 1\right) =$$

$$= \left(\frac{6}{7}, -\frac{3}{7}, -\frac{9}{7}\right)$$

reta $P_r^* + t\ \overrightarrow{P_rP_s}$

reta $P_r^* + t\ \vec{M}$ (Manoel)


v
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P_r
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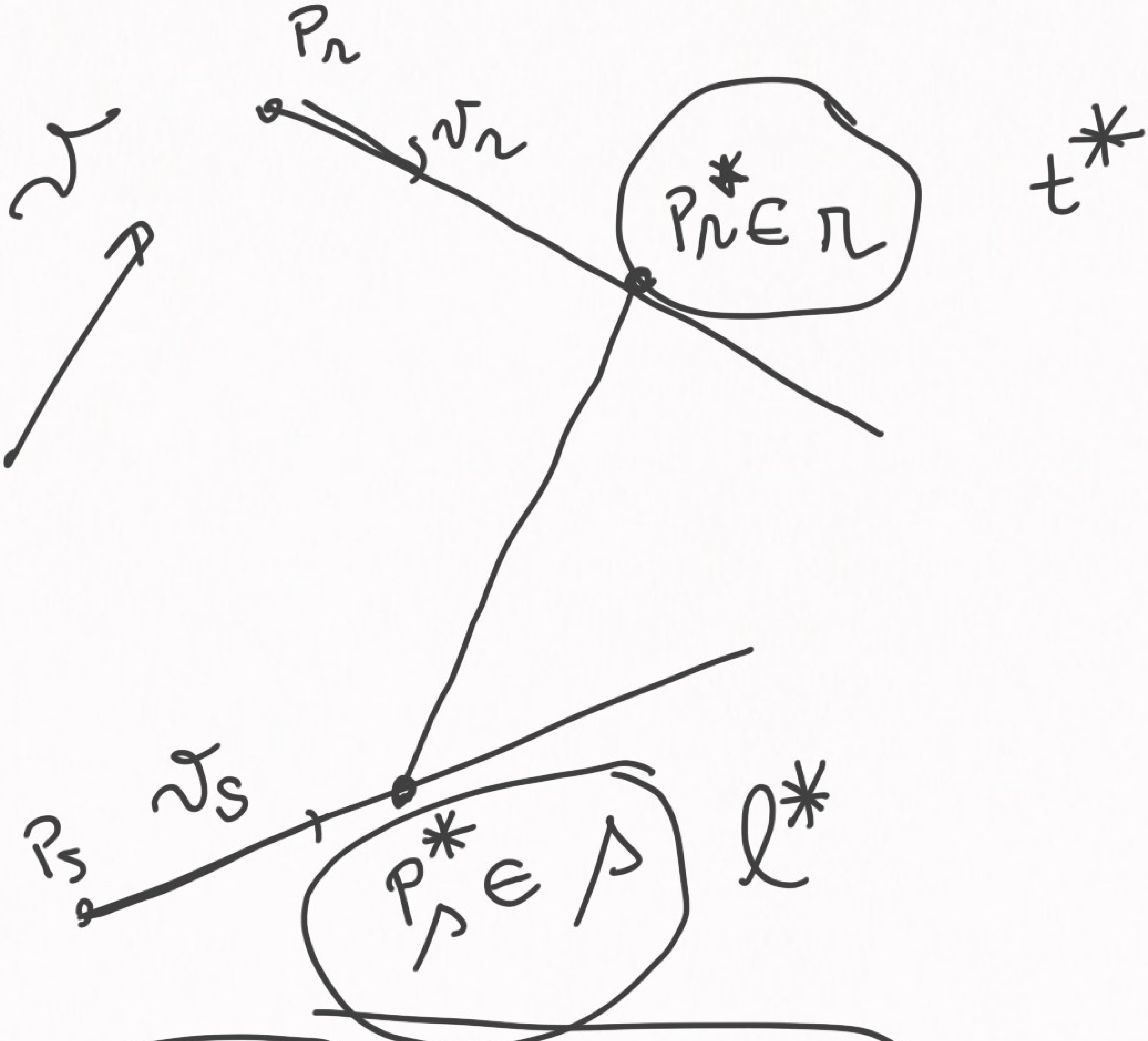


$$\boxed{\overrightarrow{P_r^* P_s^*} = \alpha \vec{v}}$$

3 eq
3 incog

1) dada

$$r: \quad P_r = (1,0,1) + t^* \overbrace{(1,2,1)}^{v_r}$$

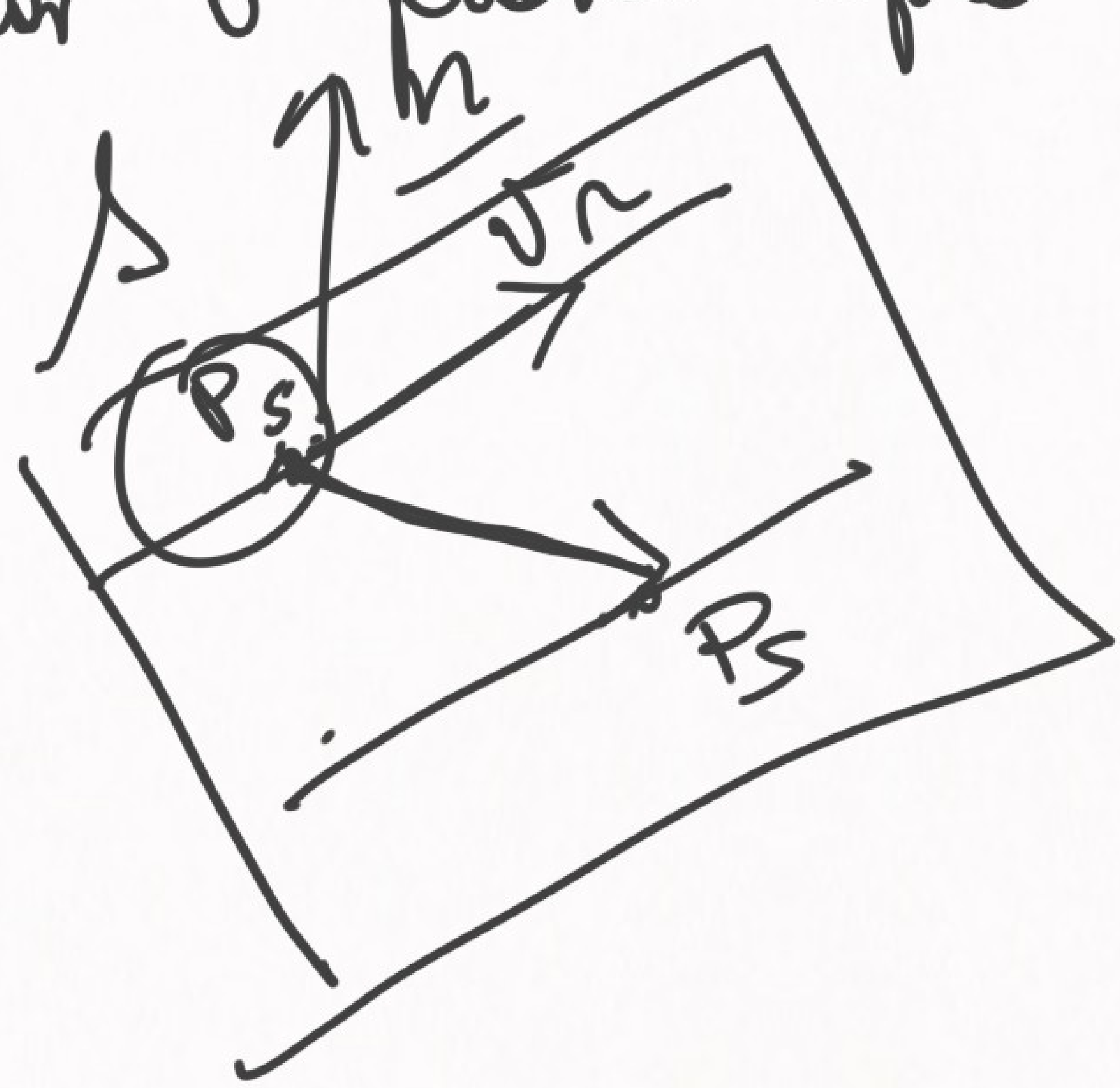
$$s: \quad P_s = (1,3,2) + t^* \underbrace{(-1,-2,-1)}_{v_s}$$

$d(P_r, P_s)$

$v_s \times v_r = 0 \Rightarrow$ paralelo

$$\Rightarrow \quad d(P_r, s) = \frac{\| \overrightarrow{P_r P_s} \times v_s \|}{\| v_s \|} \neq 0$$

$\Rightarrow$ Achar o plano que contém r e s



$$r = \boxed{(1,0,0) + t(1,-1,1)} = \bigcirc$$

$$s = (1,k,1) + t(1,2,1) \neq \checkmark$$

$$= 1$$

$$r: (1,0,0) + t(1,-1,1)$$

$$\cancel{(}2,-1,1)$$

$$s = (2,-1,1) + t(3,-2,1)$$

P^*

$$\left.\begin{aligned} v_r \times v &\neq 0 \Rightarrow \\ (v_r \times v_s)\cdot\overrightarrow{P_rP_s} &= 0 \end{aligned}\right\} \Rightarrow$$